

INDIAN STATISTICAL INSTITUTE
Probability Theory II: B. Math (Hons.) I
Semester II, Academic Year 2016-17
Mid-term Exam

Date: Feb 21, 2017 Total Marks: 40 Duration: 10:00 am - 12:30 pm

- Please write your roll number on top of your answer paper.
- Show all your works and write explanations when needed. If you are using a result stated and/or proved in class, please quote it correctly.
- You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc.

1. A continuous random vector (X, Y) has a joint probability density function given by

$$f_{X,Y}(x, y) = \begin{cases} c & \text{if } y > 0, |x| + y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) (2 marks) Find c .
- (b) (4+4 = 8 marks) Compute marginal probability density functions of X and Y .

2. (10 marks) Suppose $X_1 \sim N(0, 1)$, $X_2 \sim N(0, 1)$ and X_1, X_2 are independent. Find a probability density function of $Y_1 = X_1 + X_2$.

[**Hint:** Use the random variable $Y_2 = X_1 - X_2$.]

3. Suppose $X_1 \sim \text{Gamma}(\alpha_1, \lambda)$, $X_2 \sim \text{Gamma}(\alpha_2, \lambda)$ and X_1, X_2 are independent.

- (a) (10 marks) Find a joint probability density function of $Y_1 = X_1 + X_2$ and $Y_2 = X_1/(X_1 + X_2)$.
- (b) (2 marks) Are Y_1 and Y_2 independent? Please justify your answer.
- (c) (2 marks) Can you identify the marginal distributions of Y_1 and Y_2 ?
- (d) (5+1 = 6 marks) Let Z_1, Z_2, \dots, Z_{200} be independent and identically distributed random variables with $Z_1 \sim N(0, 1)$. Find a joint probability density function of

$$\sum_{i=1}^{200} Z_i^2 \text{ and } \frac{\sum_{i=1}^{80} Z_i^2}{\sum_{i=1}^{200} Z_i^2}. \text{ Are they independent?}$$